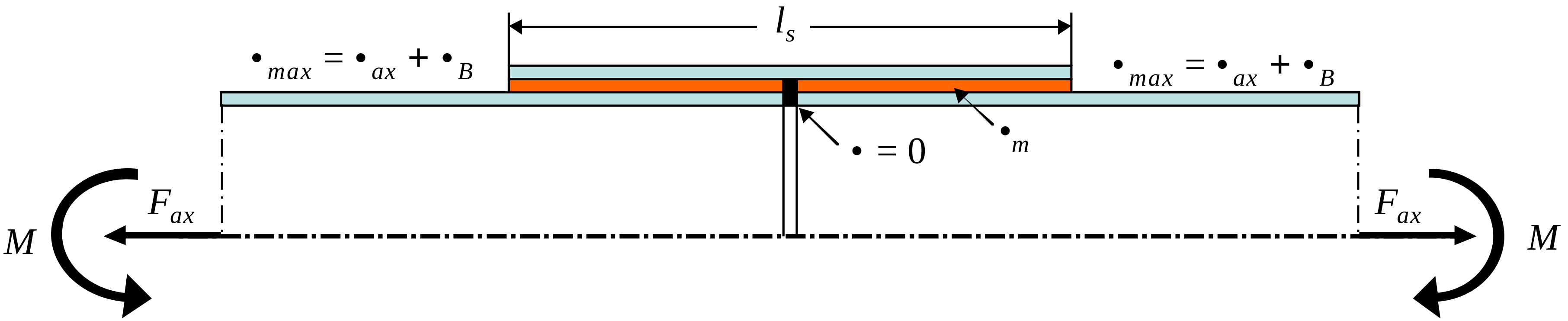
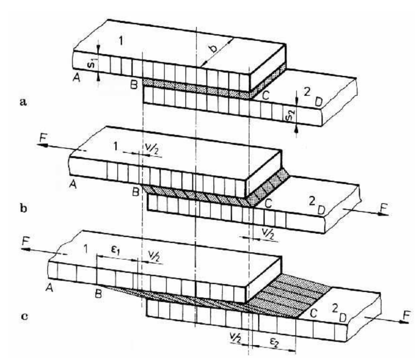
Bondline Stress Calculations

in Adhesive Joined
Pipeline Sleeve Sockets
with Volkersen‘s Equation

from Dr.-Ing. Horst Stepanski





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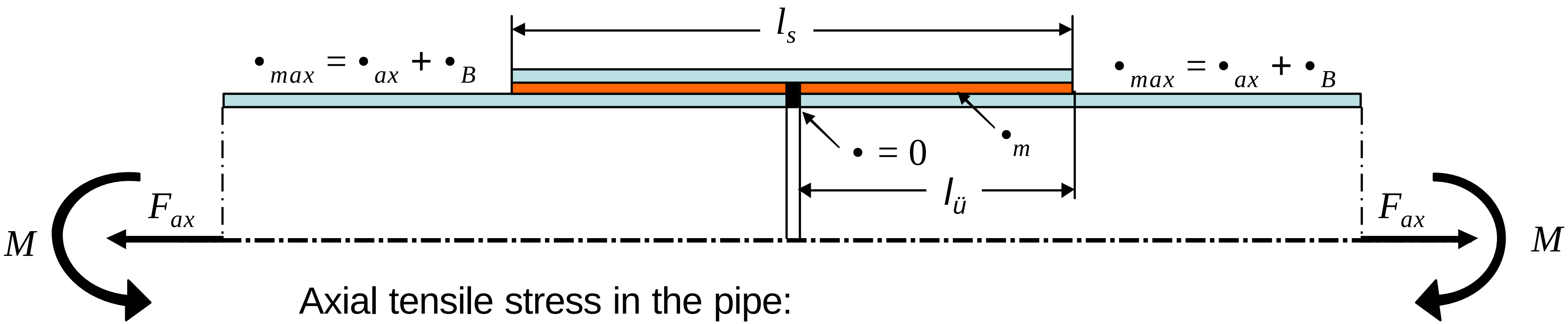
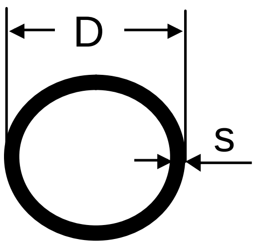
Stresses in a pipe

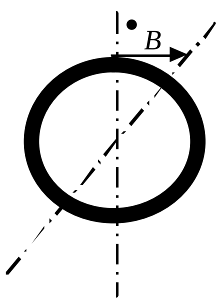




Axial tensile stress in the pipe:

$$s_{ax} = \frac{F_{ax}}{p \cdot s(D - s)}$$

Bending tensile stress in the pipe:



$$s_B = \frac{M_B}{W_{ax}} \quad \text{with} \quad W_{ax} = \frac{p d_m^2 s}{4} \quad \text{for} \quad \left(\frac{s}{d_m}\right)^2 << 1$$

Superimposed: $\bullet_{max} = \bullet_{ax} + \bullet_B$

This causes in the adhesive bondline a mean shear stress of $t_m = s_{\max} \frac{s}{l_ü}$

- **But the shear stress in the bond is non-uniform with peaks at both ends!**

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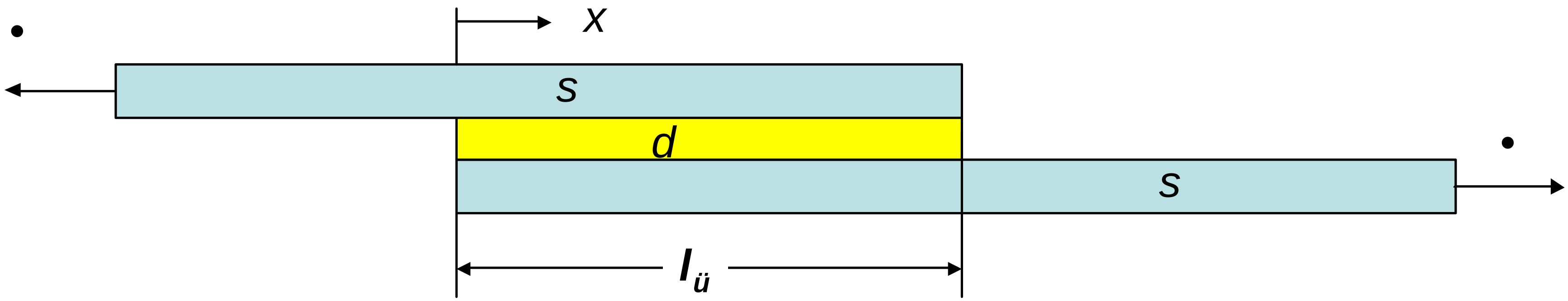
Volkersen‘s equation to calculate the shear stress peaks



Adhesive bondline lap shear stress distribution (Volkersen equation):

$$n(x)=\frac{t(x)}{t_m}=\sqrt{\frac{\Delta}{2}}\cdot\frac{\cosh\left(\sqrt{2\cdot\Delta}\cdot\frac{x}{l_{ü}}\right)+\cosh\left(\sqrt{2\cdot\Delta}\cdot\left(1-\frac{x}{l_{ü}}\right)\right)}{\sinh\left(\sqrt{2\cdot\Delta}\right)}$$

with $\Delta=\frac{G_K\cdot l_{ü}^2}{E_{St}\cdot s\cdot d}$ and $t_m=s\,\frac{s}{l_{ü}}$

The maximum shear stress at both ends of the overlap ($x = l_{ü}$ or $x = 0$) is:

$$t_{\max}=t_m\sqrt{\frac{\Delta}{2}}\coth\sqrt{\frac{\Delta}{2}}$$

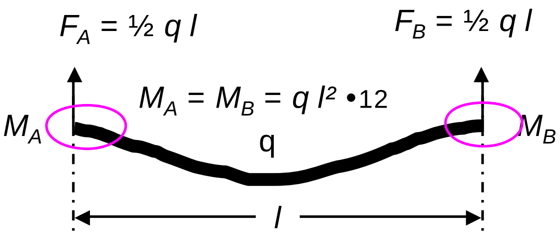
Setted values for bondline stress calculations

Pipe size	**DN 150**	**DN 500**
Outer diameter *D* in mm	168,3	508,0
Steel thickness *s* in mm	7,1	8,8
Mean diameter d_m in mm (calculated)	161,2	499,2
Adhesive‘s shear modulus in MPa	859	859
Young‘s modulus E_{St} in MPa	210 000	210 000
Yield strength •$_{0,5}$ in MPa (mean)	481	481
Sleeve length in mm	600	600
Overlapped bondline length in mm	300	300
Sleeve thickness *S* in mm	7,1	8,8
Average gap width pipe/sleeve in mm	5	5

The maximum loads occure at the construction site by the cranes lifting the pipeline into the trench:

Pipe size	DN 150	DN 500
Bending moment M_B in kNm	38,5 (58)	476,3 (717)
Corresponding crane distance l in m	41	73
Axial Force F_{ax} in kN	745 (1118)	2900 (4350)
Equ. length of vertically hanging pipe [m]	2700	2700

	Line load q
DN 150	280 N/m
DN 500	1080 N/m



The test conditions are required to be 50% higher (values in brackets)!

The test pressures should bei 50% higher compared to an operating pipeline:

Pipe size	DN 150	DN 500
Operating pressure p_{op} in bar	16	40
Test pressure p_{test} in bar	(24)	(60)

The axial force resulting from the pressure can be calculated by

$$F_{ax} = \frac{\mathrm{p} \cdot p \cdot (D - 2s)^2}{4}$$

Rohr-Nennweite	DN 150	DN 500
Axial force at operating pressure in kN	29,8	755,5
Axial force at test pressure in kN	(44,7)	(1 133,3)

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Results of stress calculations:

Pipe size	**DN 150**		**DN 500**	
Conditions	**Operation**	**Test**	**Operation**	**Test**
Bending stress •$_{B}$ in MPa	266	399	277	415
Axial tensile stress •$_{ax}$ in MPa	207	311	210	315
Superimposed stress •$_{max}$ in MPA	**473**	**710**	**487**	**730**
Shear stress mean •$_{m}$(•$_{max}$) in MPa	11	17	14	21
Shear stress peak factor (Volkersen) •$_{max}$/•$_{m}$	2,33	>> 3	2,12	>> 3
Shear stress peak (Volkersen) •$_{max}$ in MPa	**25,6**	**>> 30**	**29,7**	**>> 30**
Tensile stress caused by pressure in MPa	8	12	55	82
Average shear stress caused by pressure in MPa	0,2	0,3	1,6	2,4
Shear stress peak caused by pressure in MPa	0,4	0,7	3,4	5,1

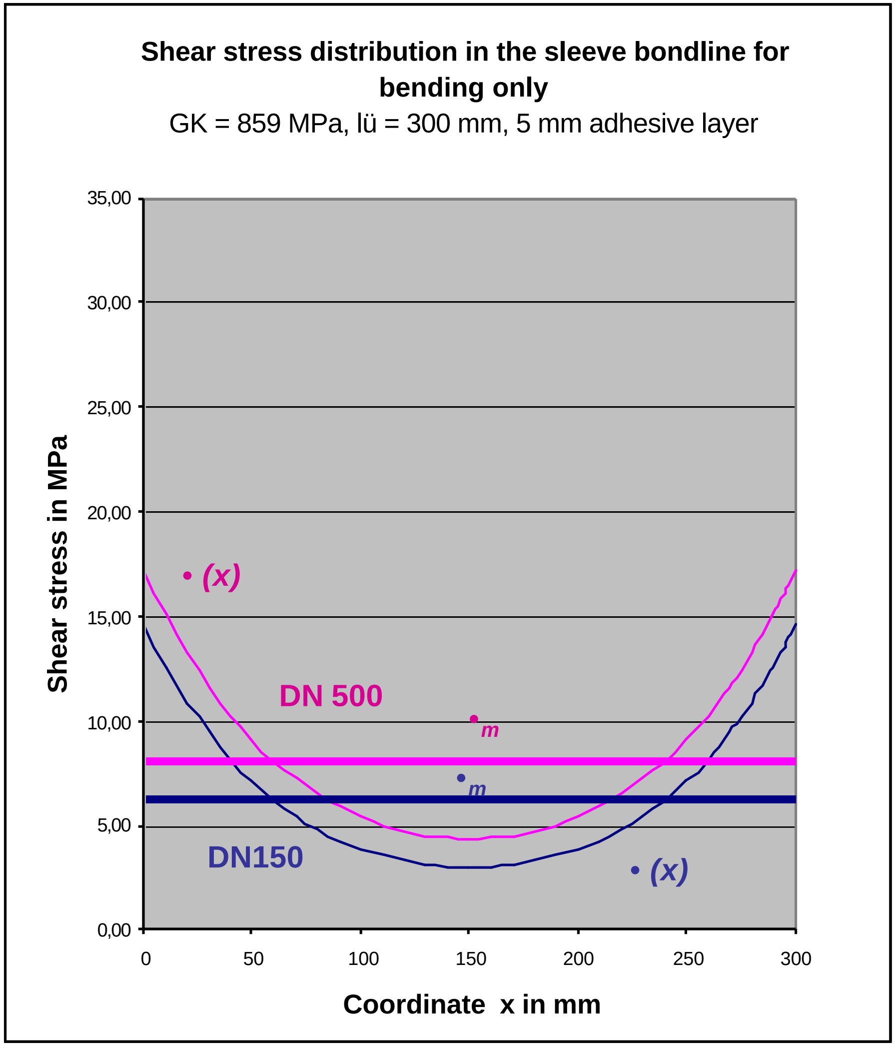
- Superimposed stresses caused by bending moment and axial force are equivalent to the yield strength of the steel (or even higher under testing conditions!)

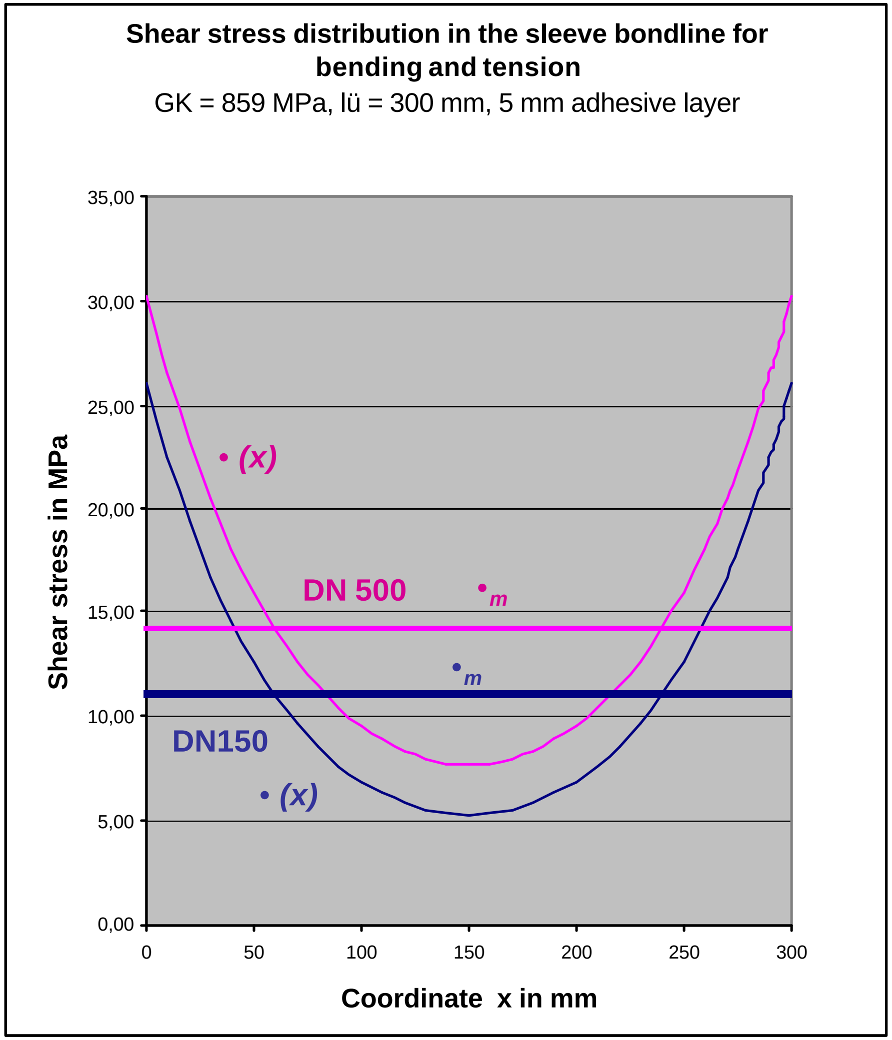
- Adhesive shear stress peaks calculated with Volkersen‘s equation are so high that adhesive is likely to fail under these conditions

- It is generally known that adhesives definitely fail in lap shear conditions when the substrats are yielding!

Lap shear stress distribution in the adhesive bond line between sleeve and pipe if axial force and bending moment are superimposed

(for loads as estimated to happen on construction site)





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Does it make sense do make the sleeve longer?

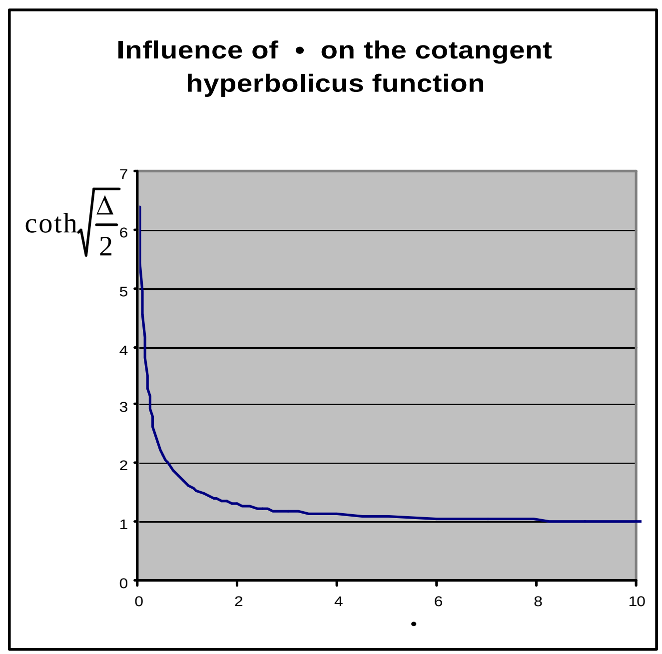
(1) $t_{max} = t_m \sqrt{\frac{\Delta}{2}} \coth \sqrt{\frac{\Delta}{2}}$ **(2)** $\Delta = \frac{G \cdot l_{ü}^2}{E \cdot s \cdot d}$ **(3)** $t_m = s \frac{s}{l_{ü}}$

$$t_m \sqrt{\frac{\Delta}{2}} = s \frac{s}{l_{ü}} \sqrt{\frac{G \cdot l_{ü}^2}{2 \cdot E \cdot s \cdot d}} = s \sqrt{\frac{G \cdot l_{ü}^2 \cdot s^2}{2 \cdot E \cdot s \cdot d \cdot l_{ü}^2}} = s \sqrt{\frac{G \cdot s}{2 \cdot E \cdot d}} \neq f(l_{ü})$$

Only the coth-term is influenced by the length of the overlap or the sleeve!

The values for $\coth \sqrt{\frac{\Delta}{2}}$

are 1,0001 for DN 150
and 1,0002 for DN 500.



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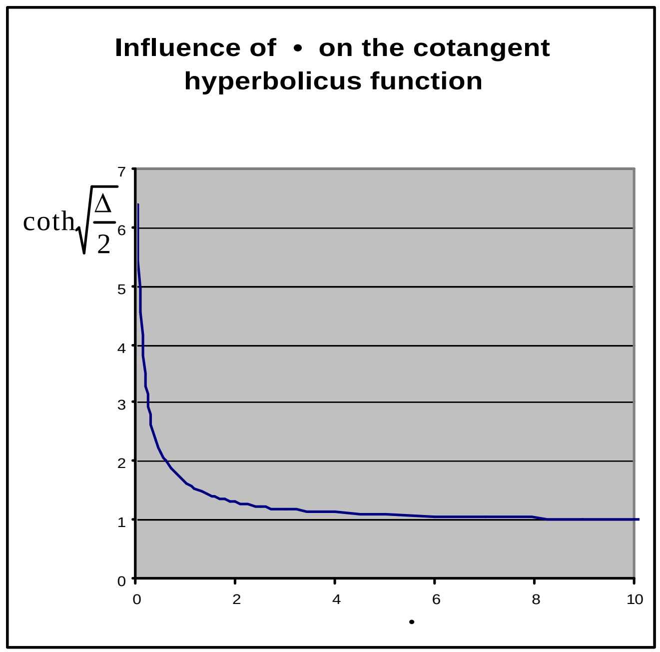
NO!



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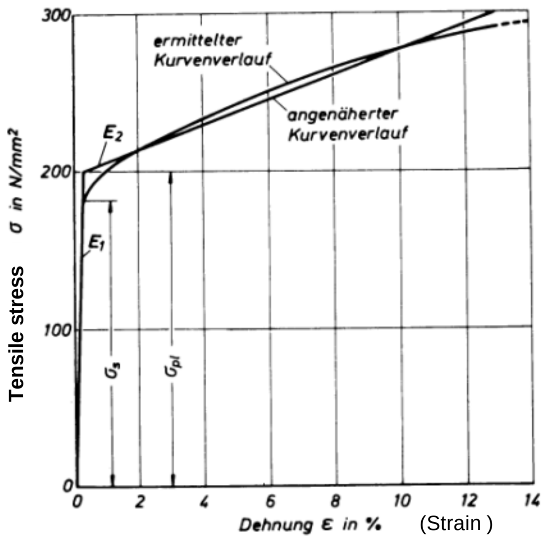
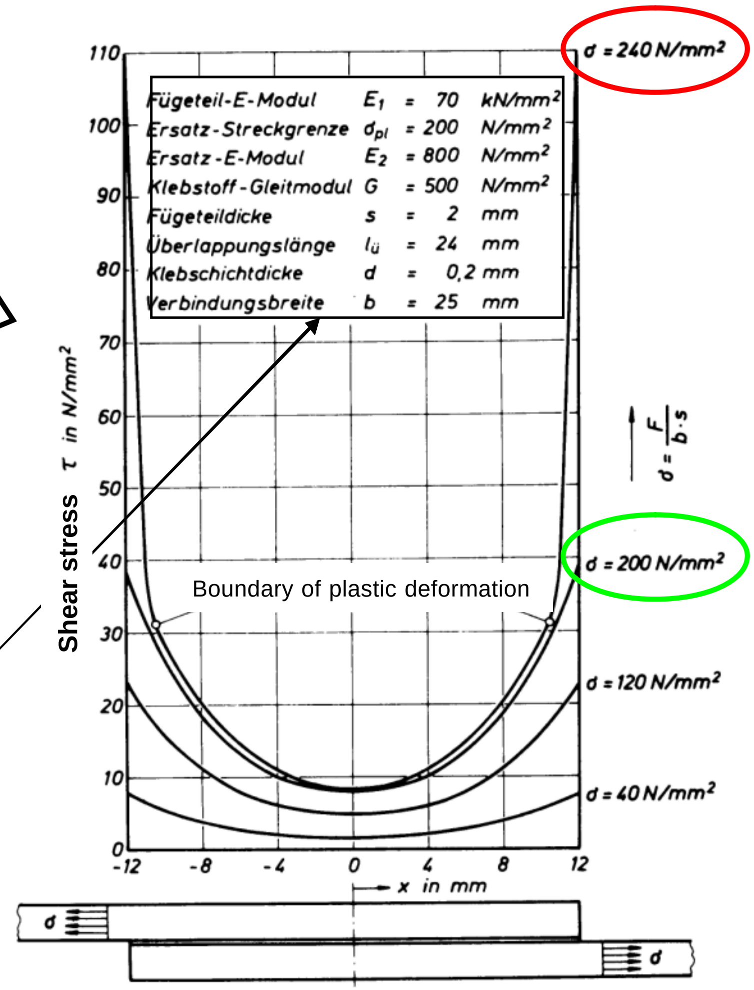
Stresses in adhesive bonded metal joints must not exceed substrate‘s yield strength!



Stress-strain diagramm of an alumium alloy and approximated bi-elastic behaviour

calculation

Shear stress distribution in an aluminium lap shear joint before and after transgressing the yield strength of the Substrate



Young‘s modulus aluminium	**E_1 =**	**70 kN •mm²**
Surrogate yield strength	**$•_{pl}$ =**	**200 N •mm²**
Surrogate yield modulus	**E_2 =**	**800 N •mm²**
Adhesive‘s shear modulus	**G =**	**500 N •mm²**
Substrate thickness	**s =**	**2 mm**
Overlap length	**l =**	**24 mm**
Adhesive thickness	**d =**	**0,2 mm**

- **substrate stress exceeding the yield strength by 20 % increases the adhesive shear stress peak by • 300 %**

Source:
Hahn, Otto, Stepanski:
Bedeutung der Fügeteilbeanspruchung für die Dimensionierung einschnittig überlappter Metallverbindungen.
Industrieanzeiger 98. Jg. Nr. 88 v. 3.11.1976, S. 1571 ff

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